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Type 0 strings in a 2-d black hole

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Abstract: We study some aspects of type 0 strings propagating in the two dimensional black hole geometry, corresponding to the exact SL(2)/ U(1) SCFT background.

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Contents

1. Introduction

In this work we study some aspects of the type 0 fermionic string[1] in a two dimensional black hole geometry.[2] The two dimensional black hole background corresponds to an exact (to all orders in the inverse string tension α') superconformal field theory (SCFT) on the worldsheet: the SL(2)/ U(1) quotient SCFT. This is the supersymmetric worldsheet version of the bosonic SL(2)/ U(1) black hole [3, 4]. In the euclidean case, the axially gauged quotient sigma model is a semi-infinite "cigar" [5] with a non-trivial dilaton. Far from the tip of the cigar, the worldsheet theory is a sigma model on the infinite cylinder with a linear dilaton in the non-compact direction, and two free fermions. The string theory is weakly coupled in this asymptotic regime. On the other hand, the maximal value of the dilaton is at the tip of the cigar; it is a finite, free parameter. Hence, this theory can be studied in a small string coupling (g_s) perturbation theory.

[1]See [1] and references therein for some background.
[2]See [2] and references therein for a review.

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The two dimensional lorentzian background can be obtained either by gauging a different U(1) in SL(2), or by an analytic continuation of the euclidean energy on the cigar. This Wick rotation maps the cigar to the regime outside the horizon of the $1+1$ dimensional black hole. The regime behind the singularity can be obtained by analytic continuation of an axial/vector T-dual version of the cigar — the "trumpet" [6, 7, 4] — or by considering the analytic continuation of correlators of winding modes on the cigar to lorentzian space-time.

Like the 2-d bosonic string, the closed string physical spectrum of the 2-d type 0 strings consists of massless "tachyons" (there are no string excitations in a two dimensional target space-time).[3] Hence, the type 0 2-d black hole is perturbatively stable. Moreover, unlike the 2-d bosonic black hole but as in [8, 9], the type 0 black hole is also expected to be non-perturbatively well defined. This fact on its own is already a good motivation to reconsider strings propagation in a 2-d black hole background, this time fermionic strings.

Another related motivation for this work is that similar to [8, 9], it is plausible that the type 0 2-d black hole has an open string dual: a matrix quantum mechanics on its (unstable) localized D-branes. We shall speculate on a possible direction to search for such a matrix model dual later.

The purpose of this work is to begin to collect some properties of the 2-d type 0 string theories on SL(2)/U(1), and to consider its black hole interpretation. In section 2, we present some properties of the SL(2)/U(1) SCFT which will be useful later. In section 3, we present physical operators in the type 0A and type 0B string theories. There are massless scalars ("tachyons") both in the NS-NS and R-R sectors of these theories. In section 4, we compute the two point functions of these scalars. Then, in section 5, we interpret these results as the reflection coefficients of scattering waves, which are incoming from an asymptotically flat regime of the black hole, either outside the horizon or beyond its singularity, and scattered either from the horizon or the singularity, respectively.

In section 6, we present our conjecture regarding a matrix quantum mechanics dual to the type 0A string theory on the 2-d black hole. Finally, in the appendices we describe some of the technical details.

2. The SL(2)/U(1) SCFT

In this section we collect some properties of the two dimensional superconformal field theory (SCFT) on SL(2)/U(1) which will be useful later (see [1] for some preliminary background and references therein, as well as [10] and references therein).

Consider first the superconformal WZW model on $SL(2)_\kappa$ at level κ. This theory has affine left-moving SL(2) supercurrents,

$$\psi^a + \theta\sqrt{\frac{2}{\kappa}}J^a\,, \qquad a = 1,2,3\,, \tag{2.1}$$

[3] There are also discrete physical states appearing at special values of momenta, which we shall not consider in this work.

where

$$J^a = j^a - \frac{i}{2}\epsilon^a{}_{bc}\psi^b\psi^c\,, \tag{2.2}$$

and

$$\begin{aligned}\psi^a(z)\psi^b(z') &\sim \frac{\eta^{ab}}{z-z'}\,, \qquad \eta^{ab} = \mathrm{diag}(+,+,-)\,,\\ J^a(z)J^b(z') &\sim \frac{\frac{\kappa}{2}\eta^{ab}}{(z-z')^2} + \frac{i\epsilon^{ab}{}_cJ^c}{z-z'}\,.\end{aligned} \tag{2.3}$$

The purely bosonic currents j^a generate an affine SL(2) algebra at level $\kappa_B = \kappa+2$ and commute with the free fermions ψ^a, whereas the total (physical) currents J^a generate a level κ SL(2) algebra and act on ψ^a as follows from (2.2), (2.3). Similarly, the theory has affine right-moving SL(2) supercurrents, $\bar\psi^a + \bar\theta\sqrt{\frac{2}{\kappa}}\bar J^a$. This theory has a Virasoro central charge

$$c(\mathrm{SL}(2)) = \frac{9}{2} + \frac{6}{\kappa}\,. \tag{2.4}$$

Define

$$\psi^\pm = \frac{1}{\sqrt{2}}(\psi^1 \pm i\psi^2)\,, \tag{2.5}$$

and introduce a canonically normalized scalar H, $H(z)H(z') \sim -\log(z-z')$, which is used to bosonize $\psi^{1,2}$:

$$\partial H \equiv \psi^2\psi^1 = i\psi^-\psi^+\,. \tag{2.6}$$

Note that

$$J^3 = j^3 + i\partial H\,, \tag{2.7}$$

and

$$j^3(z)\partial H(z') \sim 0\,, \qquad J^3(z)i\partial H(z') \sim \frac{1}{(z-z')^2}\,. \tag{2.8}$$

We introduce two other canonically normalized, independent scalars X_3 and X_R, defined by

$$J^3 = -\sqrt{\frac{\kappa}{2}}\partial X_3\,, \tag{2.9}$$

and

$$iH = \sqrt{\frac{2}{\kappa}}X_3 + i\sqrt{\frac{c}{3}}X_R\,, \tag{2.10}$$

where

$$c = c\left(\frac{\mathrm{SL}(2)}{\mathrm{U}(1)}\right) = 3 + \frac{6}{\kappa}\,. \tag{2.11}$$

The X_3 dependence in eq. (2.10) follows from (2.8) and (2.9). Equations (2.7), (2.9) and (2.10) imply that

$$j^3 = -\sqrt{\frac{\kappa+2}{2}}\partial x_3\,, \tag{2.12}$$

where the canonically normalized scalar x_3 is:

$$x_3 = \sqrt{\frac{2}{\kappa}}\left(\sqrt{\frac{\kappa+2}{2}}X_3 + iX_R\right)\,. \tag{2.13}$$

Let $\Phi_{jm\bar{m}}$ be a primary field in the bosonic WZW model on $\mathrm{SL}(2)_{\kappa_B}$ at level

$$\kappa_B = \kappa + 2\,, \tag{2.14}$$

which obeys

$$j^3(z)\Phi_{jm\bar{m}}(z') \sim \frac{m\Phi_{jm\bar{m}}(z')}{z-z'}\,, \tag{2.15}$$

and similarly $\bar{m}$ is the eigenvalue of the right-moving current $\bar{j}^3$. Since $\Phi_{jm\bar{m}}$ is purely bosonic — independent of the free fermions ψ^a — it is also a primary of the superconformal theory on $\mathrm{SL}(2)_\kappa$ at level κ with a J^3 eigenvalue equals to m:

$$J^3(z)\Phi_{jm\bar{m}}(z') \sim \frac{m\Phi_{jm\bar{m}}(z')}{z-z'}\,, \tag{2.16}$$

and similarly for the right-movers.

We denote by $\Phi_{jm}(z)$ the holomorphic part of $\Phi_{jm\bar{m}}$. Its scaling dimension is:

$$h(\Phi_{jm}) = -\frac{j(j+1)}{\kappa}\,. \tag{2.17}$$

Using eqs. (2.9) and (2.12), (2.13), we can decompose:

$$\Phi_{jm} = U_{jm}e^{m\sqrt{\frac{2}{\kappa}}X_3} = V_{jm}e^{m\sqrt{\frac{2}{\kappa+2}}x_3} = V_{jm}e^{i\frac{2m}{\kappa+2}\sqrt{\frac{c}{3}}X_R}e^{m\sqrt{\frac{2}{\kappa}}X_3}\,, \tag{2.18}$$

where V_{jm} is a primary of the bosonic euclidean quotient CFT on $\mathrm{SL}(2)_{\kappa_B}/\,\mathrm{U}(1)$ and U_{jm} is a primary of the superconformal quotient $\mathrm{SL}(2)_\kappa/\,\mathrm{U}(1)$. The Virasoro central charge c of the latter is given in eq. (2.11). The scaling dimensions of V and U are

$$h(V_{jm}) = -\frac{j(j+1)}{\kappa} + \frac{m^2}{\kappa+2}\,, \tag{2.19}$$

and

$$h(U_{jm}) = \frac{-j(j+1)+m^2}{\kappa}\,. \tag{2.20}$$

Similarly, using (2.10), we can decompose

$$e^{inH} = e^{n(\sqrt{\frac{2}{\kappa}}X_3 + i\sqrt{\frac{c}{3}}X_R)}\,. \tag{2.21}$$

Operators in the euclidean $\mathrm{SL}(2)_\kappa/\,\mathrm{U}(1)$ SCFT are obtained from their "parent" operators in the $\mathrm{SL}(2)_\kappa$ SCFT by eliminating the X_3 and ψ^3 dependence. Hence, primary fields in the $\mathrm{SL}(2)_\kappa/\,\mathrm{U}(1)$ SCFT take the form:

$$V^n_{jm} = V_{jm}e^{i(\frac{2m}{\kappa+2}+n)\sqrt{\frac{c}{3}}X_R}\,. \tag{2.22}$$

Their scaling dimension is:

$$h(V^n_{jm}) = -\frac{j(j+1)}{\kappa} + \frac{m^2}{\kappa+2} + \frac{c}{6}\left(n + \frac{2m}{\kappa+2}\right)^2 = \frac{-j(j+1)+(m+n)^2}{\kappa} + \frac{n^2}{2}\,. \tag{2.23}$$

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In the Neveu-Schwarz (NS) sector n is an integer while in the Ramond (R) sector n is in $\mathbb{Z}+1/2$:[4]

$$\begin{aligned} NS: &\qquad n \in \mathbb{Z}\,, \\ R: &\qquad n \in \mathbb{Z}+\frac{1}{2}\,. \end{aligned} \tag{2.24}$$

A few comments are in order:

1. The SCFT on $\mathrm{SL}(2)_\kappa/\,\mathrm{U}(1)$ has an $N=(2,2)$ superconformal symmetry. The holomorhic U(1) R-current of the left-moving $N=2$ algebra is

$$J_R = i\sqrt{\frac{c}{3}}\partial X_R\,, \tag{2.25}$$

and similarly for the right-moving algebra.

2. Equations (2.25) and (2.22) imply that the R-charge of V^n_{jm} is:

$$Q_R(V^n_{jm}) = \frac{c}{3}\left(\frac{2m}{\kappa+2}+n\right) = \frac{2m}{\kappa}+\frac{nc}{3}\,. \tag{2.26}$$

3. When $n=0$:

$$V^0_{jm} = U_{jm} = V_{jm}e^{i(\frac{2m}{\kappa+2})\sqrt{\frac{c}{3}}X_R}\,, \tag{2.27}$$

where V_{jm} and U_{jm} are given above (see eqs. (2.18), (2.19), (2.20), (2.22)).

4. The "parent" operator of V^n_{jm} is the $\mathrm{SL}(2)_\kappa$ operator[5]

$$\Phi_{jm}e^{inH} = V^n_{jm}e^{\sqrt{\frac{2}{\kappa}}(m+n)X_3}\,. \tag{2.28}$$

Hence, its J^3 eigenvalue is $m+n$.

5. The lorentzian $\mathrm{SL}(2)_\kappa/\,\mathrm{U}(1)$ quotient is obtained by gauging a space-like direction instead of the time-like direction generated by J^3. Hence, comment 4 implies that the analytic continuation from the euclidean black hole to the lorentzian one is done by a Wick rotation of the real $m+n$ eigenvalue to an imaginary value:

$$\begin{aligned} \text{Euclidean:}\quad m+n &= \frac{K}{2}\,, \qquad K \in R\,, \\ \text{Lorentzian:}\quad m+n &= \frac{iP}{2}\,, \qquad P \in R\,. \end{aligned} \tag{2.29}$$

6. The operators obtained by setting $n=\pm 1/2$ and taking $V_{jm}\to 1$ (which has $j=m=0$) on the right hand side of eq. (2.22) are the spin fields:

$$\mathcal{S}^{\pm} = e^{\pm\frac{1}{2}\int J_R} = e^{\pm\frac{i}{2}\sqrt{\frac{c}{3}}X_R}\,. \tag{2.30}$$

These generate the Ramond ground states when acting on the NS vacuum.

[4]This follows, for instance, from the fact that the NS (R) sector in a quotient SCFT is induced from the NS (R) sector of the "parent" theory, and from comment 4 below. It also follows from comment 6 bellow.

[5]In the R sector ($n\in\mathbb{Z}+1/2$), we have ignored the spin field s^3 of the ψ^3 fermion in SL(2), which is eliminated anyway in the quotient SCFT.

7. The geometry of the euclidean $\mathrm{SL}(2)_k/\,\mathrm{U}(1)$ SCFT background is the following [5, 3]. The metric and dilaton are[6]

$$ds^2 = \kappa(dr^2 + \tanh^2 r d\theta^2)\,, \qquad r \geq 0\,, \quad \theta \simeq \theta + 2\pi\,, \tag{2.31}$$

and

$$e^{\mathbf{\Phi}} = \frac{e^{\mathbf{\Phi}_0}}{\cosh r}\,, \tag{2.32}$$

where the dilaton $\mathbf{\Phi}$ is normalized such that the string coupling is $g_s = e^{\langle \mathbf{\Phi} \rangle}$, and $\mathbf{\Phi}_0$ is a constant. This is the cigar shaped background, which is exact to all orders in α' (up to an overall constant normalization) [11].

8. Asymptotically away from the tip of the cigar this background becomes the cylinder $R_\phi \times S^1_x$ (parametrized by two canonically normalized scalars x and ϕ) with two free fermions ψ_ϕ, ψ_x and a linear dilaton:

$$ds^2 = \frac{1}{2}(d\phi^2 + dx^2)\,, \qquad \mathbf{\Phi} = -\frac{Q}{2}\phi\,, \tag{2.33}$$

where[7]

$$Q = \sqrt{\frac{2}{\kappa}}\,, \qquad \phi = \frac{2}{Q}r\,, \qquad x = \frac{2}{Q}\theta\,, \qquad x \simeq x + \frac{4\pi}{Q}\,. \tag{2.34}$$

In the asymptotic cylinder the central charge reads:

$$c = 3(1 + Q^2)\,, \tag{2.35}$$

and the $\mathrm{U}(1)_R$ current of the $N = 2$ algebra is:

$$J_R = i\partial(H' + Qx)\,, \tag{2.36}$$

where the canonically normalized scalar H' is used to bosonize the free fermions:

$$\partial H' = \psi_x \psi_\phi\,. \tag{2.37}$$

9. The asymptotic behavior of the SCFT primaries U_{jm} is:

$$U_{jm} \to e^{iQmx} e^{Qj\phi}\,. \tag{2.38}$$

Its R-charge is thus $Q_R = Q^2 m = 2m/\kappa$ (in agreement with eqs. (2.26), (2.27)). Moreover, eqs. (2.25) and (2.36) imply that the asymptotic behavior of the scalar X_R is:

$$\sqrt{\frac{c}{3}} X_R \to H' + Qx\,, \tag{2.39}$$

[6] Here we work with $\alpha' = 2$.

[7] In the particular case $Q = 2$ ($\kappa = \frac{1}{2}$) we have: $\phi = r$ and $x = \theta$. Incidentally, we shall restrict to this particular case in the next sections.

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and thus, using eqs. (2.22), (2.27), (2.38) and (2.39), one finds that the asymptotic behavior of the other SL(2)/ U(1) superconformal primaries is:[8]

$$V^n_{jm} \to e^{inH'} e^{iQ(m+n)x} e^{Qj\phi} \,. \qquad (2.40)$$

Its scaling dimension is $h(j,m,n) = \frac{Q^2}{2}\left(-j(j+1) + (m+n)^2\right) + \frac{n^2}{2}$ (in agreement with eq. (2.23)); here we used:

$$h(e^{\alpha\phi}) = -\frac{1}{2}\alpha(\alpha+Q) \,. \qquad (2.41)$$

10. In the euclidean quotient:

$$m+n = \frac{1}{2}(p+w\kappa) \,, \qquad \bar{m}+\bar{n} = -\frac{1}{2}(p-w\kappa) \,, \qquad p, w \in \mathbb{Z} \,, \qquad (2.42)$$

where p and w are momenta and winding numbers on the cigar geometry.[9] This is manifest, for instance, from the asymptotic behavior (2.40): eqs. (2.40) and (2.42) imply that the left and right-moving momenta on the cigar are

$$(k,\bar{k}) = Q(m+n, \bar{m}+\bar{n}) = \frac{1}{\sqrt{2}}\left(\frac{p}{\sqrt{\kappa}} + w\sqrt{\kappa}, -\frac{p}{\sqrt{\kappa}} + w\sqrt{\kappa}\right),$$

which is compatible with the fact that the asymptotic radius of the cigar is

$$\frac{R}{\sqrt{\alpha'}} = \sqrt{\kappa} \,.$$

11. Unitarity implies [10, 13] that either

$$-\frac{1}{2} < j < \frac{k-1}{2} \,, \qquad j \in R \,, \qquad (2.43)$$

corresponding to states (in the discrete series) localized near the tip of the cigar, or

$$j \in -\frac{1}{2} + is \,, \qquad s \in R \,, \qquad (2.44)$$

corresponding to scattering states (in the continuous series).

[8] More precisely, $V^{n,\bar{n}}_{j,m,\bar{m}} \to e^{i(nH'-\bar{n}\bar{H}')} e^{iQ[(m+n)x-(\bar{m}+\bar{n})\bar{x}]}\left(e^{Qj\phi} + R(j,m,\bar{m})e^{-Q(j+1)\phi}\right)$ (see [12] and references therein), and it is the "reflection coefficient" $R(j,m,\bar{m})$ that we shall actually compute later.

[9] The reason that we have $(m+n, \bar{m}+\bar{n}) = \frac{1}{2}(p+w\kappa, -p+w\kappa)$ is due to the fact that the cigar is obtained by an *axial* U(1) gauging of SL(2) [4]; $(m+n, \bar{m}+\bar{n})$ are the $(J^3, \bar{J}^3)$ eigenvalues. A vector gauging leads to the T-dual trumpet [6, 7, 4], where $\bar{m}+\bar{n} \to -(\bar{m}+\bar{n})$. As a consequence, our conventions regarding type 0A versus 0B for the cigar are T-dual to what is used in Liouville theory.

3. Physical operators in type 0 string theory on SL(2)/ U(1)

The fermionic string[10] has a critical central charge $c = 15$. Hence, consistency of a fermionic string propagating in an SL(2)/ U(1) SCFT background requires that (see eq. (2.11)):

$$\kappa = \frac{1}{2} \quad \Leftrightarrow \quad c\left(\frac{\mathrm{SL}(2)_{1/2}}{\mathrm{U}(1)}\right) = 15\,. \tag{3.1}$$

We thus consider the fermionic string on the $\frac{\mathrm{SL}(2)_{1/2}}{\mathrm{U}(1)}$ superconformal worldsheet theory.

Using the results of the previous section, and standard arguments applied to a fermionic string theory in a two dimensional space-time [14, 1], one finds that the physical operators have the following form (we first consider the holomorhic part of the operators).[11] In the NS sector:

$$T^{\pm}(k) = e^{-\varphi} U_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2}} = e^{-\varphi} V_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2}} e^{i\frac{2k}{\sqrt{5}}X_R}\,, \tag{3.2}$$

where $T^+(k)$ obey the unitarity bound (2.43) when $k > 0$ and $T^-(k)$ obey the bound when $k < 0$. In the euclidean case (see eq. (2.42) with $\kappa = \frac{1}{2}$, $m = \frac{k}{2}$ and $n = 0$):

$$k = p + \frac{w}{2}\,, \qquad p, w \in \mathbb{Z}\,, \tag{3.3}$$

while in the lorentzian case:

$$k = iP\,, \qquad P \in R\,. \tag{3.4}$$

We chose to write the NS operators (3.2) in the -1 picture; φ is the scalar field arising in the bosonized superghost system of the worldsheet supersymmetry. In the R sector:

$$S^{\pm}(k) = e^{-\frac{\varphi}{2}} V^{\pm\frac{1}{2}}_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2}\mp\frac{1}{2}} = e^{-\frac{\varphi}{2}} V_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2}\mp\frac{1}{2}} e^{\frac{i}{\sqrt{5}}(2k\pm\frac{1}{2})X_R}\,. \tag{3.5}$$

Again, $S^+(k)$ obey the unitarity bound (2.43) when $k > 0$ and $S^-(k)$ obey the bound when $k < 0$, k is given in (3.3) or (3.4) (see eq. (2.42) with $\kappa = 1/2$, $m = k/2 \mp 1/2$ and $n = \pm 1/2$), and we chose to write the R sector operators in the $-1/2$ picture.

A few comments are in order:

(i) One can verify that

$$h(T^{\pm}(k)) = h(S^{\pm}(k)) = 1\,, \tag{3.6}$$

by using (2.20), (2.23) and

$$h(e^{q\varphi}) = -\frac{1}{2}q(q+2)\,. \tag{3.7}$$

(ii) The asymptotic behavior of these vertex operators at the weak coupling regime is (see also [15]):

$$T^{\pm}(k) \to e^{-\varphi} e^{ikx} e^{(-1\pm k)\phi}\,, \tag{3.8}$$

[10] For some background needed for this section see [1] and references therein.

[11] The way the left-moving part is combined with the right-moving part depends on which type of fermionic string is being considered; this will be discussed below.

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where ϕ is a canonically normalized scalar with a background charge $Q = 2$ ($\mathbf{\Phi} = -\phi$) and x is a canonically normalized scalar with periodicity:

$$x \simeq x + 2\pi \,, \tag{3.9}$$

and

$$S^{\pm}(k) \to e^{-\frac{\varphi}{2}} e^{\pm\frac{i}{2}H'} e^{ikx} e^{(-1\pm k)\phi} \,. \tag{3.10}$$

Here we used $Q = 2$ and $c = 15$ in eqs. (2.34), (2.38), (2.39) and (2.40).

(iii) The operators (3.8) and (3.10) are BRST invariant. This is shown in appendix A.

(iv) The $N = 2$ R-current is

$$J_R = i\sqrt{5}\partial X_R \to i\partial(H' + 2x) \,, \tag{3.11}$$

and the spin fields are:

$$\mathcal{S}^{\pm} = e^{\pm\frac{1}{2}\int J_R} = e^{\pm\frac{i}{2}\sqrt{5}X_R} \to e^{\pm\frac{i}{2}(H'+2x)} \,. \tag{3.12}$$

The right hand side in eqs. (3.11) and (3.12) is the asymptotic behavior.

(v) The analysis for the anti-holomorhic part of the vertex operators is obvious, and follows the same lines as above, but now for the right-movers. In euclidean space-time the left and right-moving "momenta" on the cigar are (see eq. (2.42) and use the same arguments which led to (3.3)):

$$(k, \bar{k}) = \left(p + \frac{w}{2}, -p + \frac{w}{2}\right) , \qquad p, w \in \mathbb{Z} \,. \tag{3.13}$$

(vi) Plugging the values $j = -\frac{1}{2} + i\frac{P}{2}$ (corresponding to scattering states (2.44)) and $m+n = i\frac{E}{2}$ (corresponding to a Wick rotation to the lorentzian black hole (2.29)) into U_{jm} and V^n_{jm}, we see that the on-shell conditions (3.6) translate into the condition:

$$E^2 - P^2 = 0 \,. \tag{3.14}$$

Hence, after combining left-movers with right-movers, the physical vertex operators correspond to massless fields in the $1 + 1$ dimensional space-time.

We are now prepared to construct the type 0 theories. This requires to combine left-movers with right-movers in a way that is tree-level unitary and one-loop modular invariant. There are four non-equivalent ways to do it:

1. **Type 0B.** We include in the theory the two (mutually local) operators Q^{++} and Q^{--},

$$Q^{\pm\pm}(z, \bar{z}) = e^{-\frac{\varphi}{2}-\frac{\bar{\varphi}}{2}} \mathcal{S}^{\pm}\bar{\mathcal{S}}^{\pm} \,, \tag{3.15}$$

where the spin fields $\mathcal{S}^{\pm}$ are given in (3.12). Mutual locality with these operators is the analog of the diagonal GSO projection in flat space. This mutual locality implies that[12]

$$n = \bar{n}\,. \tag{3.16}$$

Hence, there are physical operators only in the NS-NS and R-R sectors. The on-shell condition further implies (see (2.23)) that

$$(m+n)^2 = (\bar{m}+\bar{n})^2\,. \tag{3.17}$$

Hence, in the NS-NS sector ($n=\bar{n}=0$) we have:

$$NS-NS: \qquad m = \pm\bar{m}\,. \tag{3.18}$$

On the other hand, in the R-R sector ($n=\bar{n}=\pm\frac{1}{2}$), eq. (3.17) now implies:[13]

$$R-R: \qquad m = \bar{m}\,. \tag{3.19}$$

All in all, we learn that the physical operators in the NS-NS sector are:

$$\mathbf{T}^{\pm\epsilon}(k) = e^{-\varphi-\bar{\varphi}}V^{0,0}_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2},\epsilon\frac{k}{2}} = e^{-\varphi-\bar{\varphi}}V_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2},\epsilon\frac{k}{2}}\,e^{i\frac{2k}{\sqrt{5}}(X_R(z)-\epsilon\bar{X}_R(\bar{z}))}\,, \qquad \epsilon=\pm\,, \tag{3.20}$$

where, in the euclidean case, $\mathbf{T}^{+\epsilon}(k)$ obey the unitarity bound (2.43) when $k>0$ and $\mathbf{T}^{-\epsilon}(k)$ obey the bound when $k<0$, and

$$\begin{aligned} k &= \bar{k} = \frac{w}{2} \in \frac{\mathbb{Z}}{2} \qquad &&\text{if} \quad \epsilon = 1\,, \\ k &= -\bar{k} = p \in \mathbb{Z} \qquad &&\text{if} \quad \epsilon = -1\,. \end{aligned} \tag{3.21}$$

The physical operators in the R-R sector are:

$$\begin{aligned} \mathbf{S}^{\pm}(k) &= e^{-\frac{\varphi}{2}-\frac{\bar{\varphi}}{2}}V^{\pm\frac{1}{2},\pm\frac{1}{2}}_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2}\mp\frac{1}{2},\frac{k}{2}\mp\frac{1}{2}} \\ &= e^{-\frac{\varphi}{2}-\frac{\bar{\varphi}}{2}}V_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2}\mp\frac{1}{2},\frac{k}{2}\mp\frac{1}{2}}\,e^{\frac{i}{\sqrt{5}}(2k\pm\frac{1}{2})(X_R(z)-\bar{X}_R(\bar{z}))}\,, \end{aligned} \tag{3.22}$$

where, in the euclidean case, $\mathbf{S}^{+}(k)$ obey the unitarity bound (2.43) when $k>0$ and $\mathbf{S}^{-}(k)$ obey the bound when $k<0$, and

$$k = \bar{k} = \frac{w}{2}\,, \qquad w \in \mathbb{Z}\,. \tag{3.23}$$

The operators $V^{n\bar{n}}_{jm\bar{m}}$ in (3.20) and (3.22) are the combinations of the left-moving primaries V^{n}_{jm} in (2.22) with the right-moving $\bar{V}^{\bar{n}}_{j\bar{m}}$. In both the NS-NS and the R-R sectors, we have:

$$m+n = \frac{k}{2}\,. \tag{3.24}$$

[12] More generally, mutual locality implies that $n-\bar{n}\in 2\mathbb{Z}$. However, since the 2-d string theory has no physical excitations, the operators $T(k)$ and $S(k)$ only have $n=0$ and $n=\pm\frac{1}{2}$, respectively.

[13] The solution $\bar{m}=-m-2n$ leads to an operator which is not BRST invariant.

On the other hand, for the right-moving $\bar{J}^3$ number we have:

$$
\begin{aligned}
\bar{m}+\bar{n} &= \pm\frac{k}{2} \qquad \text{in} \quad NS-NS\,, \\
\bar{m}+\bar{n} &= \frac{k}{2} \qquad \text{in} \quad R-R\,.
\end{aligned} \tag{3.25}
$$

Hence, the analytic continuation from the euclidean cigar to the lorentzian black hole is done by Wick rotating (see (2.29)):

$$
k \to iP\,, \qquad P \in R\,. \tag{3.26}
$$

2. **Type 0A.** We include in the theory the two (mutually local) operators Q^{+-} and Q^{-+},

$$
Q^{\pm\mp}(z,\bar{z}) = e^{-\frac{\varphi}{2}-\frac{\bar{\varphi}}{2}}\mathcal{S}^{\pm}\bar{\mathcal{S}}^{\mp}\,, \tag{3.27}
$$

where the spin fields $\mathcal{S}^{\pm}$ are given in (3.12). Mutual locality now implies that[14]

$$
n = -\bar{n}\,, \tag{3.28}
$$

and again, there are physical operators only in the NS-NS and R-R sectors. As before, the on-shell condition further implies eq. (3.17). Hence, in the NS-NS sector ($n = \bar{n} = 0$) we have again the relation (3.18), but in the R-R sector ($n = -\bar{n} = \pm 1/2$), eq. (3.17) now implies:

$$
R-R: \qquad m = -\bar{m}\,. \tag{3.29}
$$

Thus, the physical operators in the NS-NS sector are the same as in type 0B (3.20), (3.21), while in the R-R sector the physical operators are:

$$
\begin{aligned}
\tilde{\mathbf{S}}^{\pm}(k) &= e^{-\frac{\varphi}{2}-\frac{\bar{\varphi}}{2}}V^{\pm\frac{1}{2},\mp\frac{1}{2}}_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2}\mp\frac{1}{2},-\frac{k}{2}\pm\frac{1}{2}} \\
&= e^{-\frac{\varphi}{2}-\frac{\bar{\varphi}}{2}}V_{-\frac{1}{2}\pm\frac{k}{2},\frac{k}{2}\mp\frac{1}{2},-\frac{k}{2}\pm\frac{1}{2}}\, e^{\frac{i}{\sqrt{5}}(2k\pm\frac{1}{2})(X_R(z)+\bar{X}_R(\bar{z}))}\,,
\end{aligned} \tag{3.30}
$$

where, in the euclidean case,

$$
k = -\bar{k} = p\,, \qquad p \in \mathbb{Z}\,. \tag{3.31}
$$

Again, both in the NS-NS and R-R sectors eq. (3.24) is satisfied. On the other hand, for the right-moving numbers we now have:

$$
\begin{aligned}
\bar{m}+\bar{n} &= \pm\frac{k}{2} \qquad \text{in} \quad NS-NS\,, \\
\bar{m}+\bar{n} &= -\frac{k}{2} \qquad \text{in} \quad R-R\,.
\end{aligned} \tag{3.32}
$$

Therefore, the analytic continuation to the lorentzian black hole is done again by Wick rotating as in (3.26).

[14] More generally, mutual locality implies that $n + \bar{n} \in 2\mathbb{Z}$, but $T(k)$ and $S(k)$ only have $n = 0$ and $n = \pm 1/2$, respectively.

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3. **Type IIB.** Here we require mutual locality with the holomorhic operators $Q^+(z)$ and $\bar{Q}^+(\bar{z})$ (or $Q^-(z)$ and $\bar{Q}^-(\bar{z})$), where

$$Q^{\pm}(z) = e^{-\frac{\varphi}{2}}\mathcal{S}^{\pm}(z)\,, \tag{3.33}$$

and similarly for the right movers [16, 17, 15]. This is the analog of a chiral GSO projection in flat space, and it leads to the supersymmetric 2-d black hole (in the euclidean case). We shall not consider this theory here, except for two remarks: operators in the NS-NS and R-R sectors correspond to space-time bosons, while operators in the NS-R and R-NS sectors correspond to space-time fermions. However, there is no boson-fermion degeneracy in the spectrum, because there is no Poincaré invariance in space-time. Some evidence in favor of a supersymmetric matrix model dual to the superstrings in the 2-d black hole geometry was presented recently in [18].

4. **Type IIA.** Here we impose an opposite chiral projection on the left and right-movers, namely, we include the holomorhic operators $Q^+(z)$ and $\bar{Q}^-(\bar{z})$ (or $Q^-(z)$ and $\bar{Q}^+(\bar{z})$), where $Q^{\pm}(z)$ are given in (3.33). We shall not consider this theory here, except for the remarks in the type-IIB case which are correct in the IIA case as well.

4. The two point functions

The physical operators $\mathbf{T}(k)$, $\mathbf{S}(k)$ and $\tilde{\mathbf{S}}(k)$ are of the form:

$$V_{jm\bar{m};q\bar{q}}(k_R,\bar{k}_R) = e^{q\varphi+\bar{q}\bar{\varphi}}V_{jm\bar{m}}e^{i(k_R X_R-\bar{k}_R\bar{X}_R)}\,. \tag{4.1}$$

Hence, the two point functions (2-p-f) take the form:[15]

$$\begin{aligned}&\langle V_{j'm'\bar{m}';q'\bar{q}'}(k'_R,\bar{k}'_R)V_{jm\bar{m};q\bar{q}}(k_R,\bar{k}_R)\rangle\\ &\quad = \langle e^{q'\varphi+\bar{q}'\bar{\varphi}}e^{q\varphi+\bar{q}\bar{\varphi}}\rangle\langle e^{i(k'_R X_R-\bar{k}'_R\bar{X}_R)}e^{i(k_R X_R-\bar{k}_R\bar{X}_R)}\rangle\langle V_{j'm'\bar{m}'}V_{jm\bar{m}}\rangle\,.\end{aligned} \tag{4.2}$$

Unlike flat target-space, in $\mathrm{SL}(2)/\mathrm{U}(1)$ the 2-p-f on the sphere do not necessarily vanish [10, 19]. Conservation laws require though that the 2-p-f vanish unless:

$$\begin{aligned}k_R+k'_R &= 0 = \bar{k}_R+\bar{k}'_R\,,\\ q+q' &= -2 = \bar{q}+\bar{q}'\,,\\ m+m' &= 0 = \bar{m}+\bar{m}'\,, \qquad j=j'\,.\end{aligned} \tag{4.3}$$

The first two correlators on the r.h.s of eq. (4.2) are trivial: they are equal to 1 when (4.3) is satisfied. The third two point function is a 2-p-f in bosonic $\mathrm{SL}(2)/\mathrm{U}(1)$ at level $\kappa_B=\kappa+2$. When (4.3) is satisfied, it is equal to [10, 19]:[16]

$$\langle V_{j,-m,-\bar{m}}V_{jm\bar{m}}\rangle = \nu(\kappa)^{2j+1}\frac{\Gamma(1-\frac{2j+1}{\kappa})}{\Gamma(1+\frac{2j+1}{\kappa})}\frac{\Gamma(-2j-1)}{\Gamma(2j+1)}\frac{\Gamma(j+m+1)\Gamma(j-\bar{m}+1)}{\Gamma(-j-\bar{m})\Gamma(-j+m)}\,. \tag{4.4}$$

[15] We suppress the $z,\bar{z}$ dependence.

[16] Note that (in the euclidean cigar) $\langle V_{j,-m,-\bar{m}}V_{jm\bar{m}}\rangle = \langle V_{jm\bar{m}}V_{j,-m,-\bar{m}}\rangle$, as it should, since $m-\bar{m}=p\in\mathbb{Z}$; this can be verified by using properties of Γ functions on the right hand side. On the other hand, $\langle V_{-(j+1),-m,-\bar{m}}V_{-(j+1),m,\bar{m}}\rangle = \langle V_{j,-m,-\bar{m}}V_{jm\bar{m}}\rangle^{-1}$.

The $j, m, \bar{m}$-independent constant ν is the analog of the cosmological constant μ in Liouville theory (more precisely, a sine-Liouville; see eq. (4.89) in [20]). From now on we choose:[17]

$$\nu(\kappa) = 1\,. \tag{4.5}$$

We shall now use these results to compute the 2-p-f of NS-NS and R-R scalars in the type 0 string theories on the 2-d black hole background.

4.1 The 2-P-F of NS-NS scalars in type 0B and 0A

The physical spectrum of NS-NS scalars is identical in the type 0B and type 0A theories. In the euclidean cigar, it consists of states with $m = -\bar{m} = k/2 = p/2$ ($\epsilon = -$ in (3.20)) and $m = \bar{m} = k/2 = w/4$ ($\epsilon = +$ in (3.20)). Recall that the integers p and w are momentum and winding numbers on the cigar background.

For $m = \bar{m}$ it is convenient to define:

$$\begin{aligned} T(k) &= \mathbf{T}^{++}(k) \qquad k > 0\,, \\ T(k) &= \mathbf{T}^{-+}(k) \qquad k < 0\,, \end{aligned} \tag{4.6}$$

and for $m = -\bar{m}$:

$$\begin{aligned} \tilde{T}(k) &= \mathbf{T}^{+-}(k) \qquad k > 0\,, \\ \tilde{T}(k) &= \mathbf{T}^{--}(k) \qquad k < 0\,. \end{aligned} \tag{4.7}$$

Following the details in appendix B, the two point functions are:

$$\begin{aligned} k = \frac{w}{2} \in \frac{\mathbb{Z}}{2}\,, \qquad & m = \bar{m}\,: \\ \langle T(-k)T(k)\rangle \quad &= \quad \frac{\Gamma(1-2|k|)\Gamma(-|k|)\Gamma(\frac{1}{2}+|k|)}{\Gamma(1+2|k|)\Gamma(|k|)\Gamma(\frac{1}{2}-|k|)}\,, \end{aligned} \tag{4.8}$$

and[18]

$$\begin{aligned} k = p \in \mathbb{Z}\,, \qquad & m = -\bar{m}\,: \\ \langle \tilde{T}(-k)\tilde{T}(k)\rangle &= \left(\frac{\Gamma(1-2|k|)\Gamma(-|k|)}{\Gamma(1+2|k|)\Gamma(|k|)}\right)\left(\frac{\Gamma(\frac{1}{2}+k)}{\Gamma(\frac{1}{2})}\right)^{2\,\mathrm{sign}(k)} \\ &= \langle T(-k)T(k)\rangle \left(\frac{1}{\cos(\pi k)}\right)^{\mathrm{sign}(k)}\,. \end{aligned} \tag{4.9}$$

After analytic continuation $|k| \to iP$, $P \in R$, to the $1+1$ lorentzian black hole, we have:

$$(R_s(P))^{\mathrm{sign}(P)} \equiv \langle T(-iP)T(iP)\rangle = e^{i\varphi(P)}\,, \qquad P \in R\,, \tag{4.10}$$

[17] See [20] for a discussion on the freedom to make such a choice.

[18] Note that for $k = p \in \mathbb{Z}$ we have $\cos(\pi p) = 1$, hence $\langle \tilde{T}(-p)\tilde{T}(p)\rangle = \langle \tilde{T}(p)\tilde{T}(-p)\rangle = \langle T(-p)T(p)\rangle$, and there are double poles in (4.8) and here. We keep however the present form of $\langle \tilde{T}(-k)\tilde{T}(k)\rangle$ since we are interested in its analytic continuation.

where the phase $\varphi(P)$ is

$$i\varphi(P) = \log\left(\frac{\Gamma(1-2iP)\Gamma(-iP)\Gamma(\frac{1}{2}+iP)}{\Gamma(1+2iP)\Gamma(iP)\Gamma(\frac{1}{2}-iP)}\right)\,, \tag{4.11}$$

and

$$(R_h(P))^{\mathrm{sign}(P)} \equiv \langle\tilde{T}(-iP)\tilde{T}(iP)\rangle = e^{i\varphi(P)}\left(\frac{1}{\cosh(\pi P)}\right)^{\mathrm{sign}(P)}\,, \qquad P\in R\,. \tag{4.12}$$

4.2 The 2-P-F of R-R scalars in type 0B

The physical spectrum in the R-R sector of the type 0B string theory on the cigar consists of winding modes (3.23), (3.24): $m+n=\bar{m}+\bar{n}=\frac{k}{2}=\frac{w}{4}$. It is convenient to define:

$$\begin{aligned} S(k) &= \mathbf{S}^{+}(k) \qquad k>0\,,\\ S(k) &= \mathbf{S}^{-}(k) \qquad k<0\,, \end{aligned} \tag{4.13}$$

where $S(k)$ is either in the $(-1/2,-1/2)$ or the $(-3/2,-3/2)$ picture (see appendices A and C). Following the details in appendix C, the two point functions are:

$$\langle S(-k)S(k)\rangle = \frac{\Gamma(1-2|k|)\Gamma(-|k|)}{\Gamma(1+2|k|)\Gamma(1-|k|)\Gamma(0)} = \frac{2(-)^{w}}{(|w|!)^2}\,, \tag{4.14}$$

(or its inverse). The analytic continuation to scattering states in lorentzian space gives (see appendix C):

$$R_s(P)=0\,, \qquad \forall\, P\,, \tag{4.15}$$

(or its inverse).

4.3 The 2-P-F of R-R fields in type 0A

The physical spectrum in the R-R sector of the type 0A string theory on the cigar consists of momentum modes (3.31), (3.32): $m+n=-\bar{m}-\bar{n}=k/2=p/2$. It is convenient to define:

$$\begin{aligned} \tilde{S}(k) &= \tilde{\mathbf{S}}^{+}(k) \qquad k>0\,,\\ \tilde{S}(k) &= \tilde{\mathbf{S}}^{-}(k) \qquad k<0\,, \end{aligned} \tag{4.16}$$

where $\tilde{S}(k)$ is either in the $(-1/2,-1/2)$ or the $(-3/2,-3/2)$ picture (see appendices A, C and D). Following the details in appendix D, the two point functions are:

$$\langle\tilde{S}(-k)\tilde{S}(k)\rangle = \frac{2(-)^{p-1}}{(|2p|!)^2}\,, \tag{4.17}$$

(or its inverse). They are equal to the 2-p-f of momentum modes in the NS-NS sector, up to "leg factors:"

$$\langle \tilde{S}(-k)\tilde{S}(k)\rangle = \langle \tilde{T}(-k)\tilde{T}(k)\rangle \left(\frac{\Gamma(\frac{1}{2}+k)}{\Gamma(\frac{1}{2})}\right)^2 \left(\frac{\Gamma(1)}{\Gamma(1+k)}\right)^2, \qquad k<0\,, \tag{4.18}$$

$$\langle \tilde{S}(-k)\tilde{S}(k)\rangle = \langle \tilde{T}(-k)\tilde{T}(k)\rangle \left(\frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}+k)}\right)^2 \left(\frac{\Gamma(k)}{\Gamma(0)}\right)^2, \qquad k>0\,, \tag{4.19}$$

(or its inverse), where $\langle \tilde{T}(-k)\tilde{T}(k)\rangle$ is given in (4.9). Recall that actually (4.19) = (4.18) = (4.17).

The formal analytic continuation of (4.18) to scattering states in lorentzian space gives:[19]

$$(R_h(P))^{-1} \equiv \langle \tilde{S}(-iP)\tilde{S}(iP)\rangle = \langle \tilde{T}(-iP)\tilde{T}(iP)\rangle \left(\frac{\Gamma(\frac{1}{2}+iP)}{\Gamma(\frac{1}{2})}\right)^2 \left(\frac{\Gamma(1)}{\Gamma(1+iP)}\right)^2, \tag{4.20}$$

or its inverse,[20] where $\langle \tilde{T}(-iP)\tilde{T}(iP)\rangle$ is given in (4.12). On the other hand, the analytic continuation of (4.19) is 0 for all P.

5. Scattering from the black hole horizon and singularity

In this section we present the interpretation of the results in the previous section in terms of the black hole physics.

5.1 Scattering of NS-NS scalars

The two point functions are normalized such that, after analytic continuation, they give the reflection coefficients[21] $R(P)$ of scattering waves, which are incoming from an asymptotically flat regime in the extended eternal black hole.[22]

The scattering of momentum modes in the cigar background has the following interpretation. It is analytically continued to the scattering of momentum modes incoming from the asymptotically flat regime outside the black hole horizon, and scattered from the horizon. The result (4.12) shows that the reflection coefficient $R_h(P)$ satisfies:

$$|R_h(P)| = \left|\frac{\Gamma(\frac{1}{2}+iP)}{\Gamma(\frac{1}{2})}\right|^2 = \frac{1}{\cosh(\pi P)}\,. \tag{5.1}$$

Hence,

$$|R_h(P)| < 1\,, \tag{5.2}$$

namely, part of the incoming wave is reflected from the black hole horizon.

[19] For $P<0$; see appendix D and recall that we choose the branch with $-i(j+1/2)>0$.

[20] For $P>0$.

[21] Or the inverse of $R(P)$, depending on whether the scattering wave is incoming or outgoing, namely, on the relative sign of energy and momentum in the scattering state. This relative sign is changed when we take $(j,m,\bar{m}) \to (-(j+1),m,\bar{m})$: it takes $P=-2i(j+1/2)\to -P$ and does not change $E=-i(m\pm\bar{m})$ ($\pm$ sign if $m=\pm\bar{m}$; see [21, 12]).

[22] This can be shown, for instance, by following sections 3 and 4 of [12], and references therein.

On the other hand, the scattering of winding modes on the cigar geometry has a different interpretation [6]. It is analytically continued to the scattering of momentum modes incoming from the asymptotically flat regime behind the black hole singularity, and scattered from the singularity. The result (4.10) shows that the NS-NS scalars are fully reflected from the singularity:

$$|R_s(P)| = 1\,. \tag{5.3}$$

All in all, the results in this subsection show that the scattering of NS-NS scalars in the 2-d type 0 black hole is rather similar to the scattering in the bosonic string theory on the 2-d black hole [23] [4, 12], where[24]

$$\begin{aligned}
&\textit{Bosonic} \quad \frac{\mathrm{SL}(2)_{k_B=9/4}}{\mathrm{U}(1)}: \quad j = -\frac{1}{2} + i\frac{P}{2}\,, \\
&m = -\bar{m} = i\frac{3P}{2}: \quad R_h(P) = \frac{\Gamma(1-4iP)\Gamma(-iP)}{\Gamma(1+4iP)\Gamma(iP)}\left(\frac{\Gamma(\frac{1}{2}+2iP)}{\Gamma(\frac{1}{2}-iP)}\right)^2, \\
&\qquad\qquad |R_h(P)| = \frac{\cosh(\pi P)}{\cosh(2\pi P)}\,, \\
&m = \bar{m} = i\frac{3P}{2}: \quad R_s(P) = \frac{\Gamma(1-4iP)\Gamma(-iP)\Gamma(\frac{1}{2}+2iP)\Gamma(\frac{1}{2}-iP)}{\Gamma(1+4iP)\Gamma(iP)\Gamma(\frac{1}{2}-2iP)\Gamma(\frac{1}{2}+iP)}\,, |R_s(P)| = 1\,.
\end{aligned} \tag{5.4}$$

In the next subsection we discuss the scattering of R-R fields.

5.2 Scattering of R-R fields

In the type 0B string theory the R-R fields have $m = \bar{m}$ (3.19), and thus they carry only winding numbers (3.23) on the cigar. Thus the result (4.15) indicates that these R-R fields are fully transmitted through the singularity.[25] This result is puzzling, since $R_s(P) = 0$ both for the dispersion relation $E = P = -ik$ (where $E = -i(m+n+\bar{m}+\bar{n}) = -2i(m+1/2)$ is the energy and $P = -2i(j+1/2)$ is the momentum), and for $E = -P = -ik$ (see appendix C). Perhaps it indicates that these R-R scalars are non-propagating fields in the 2-d black hole background.

On the other hand, in the type 0A black hole, after a formal analytic continuation we found two possibilities. The first is a propagating R-R field (see (4.18), (4.20)), with $m = -\bar{m}$ (3.29), which thus carries only momentum modes on the cigar (3.31). This

[23] Though the 2-p-f in the bosonic and type 0 cases are *different* in a significant way, which will be important later.

[24] The exact two point function on the sphere in the 2-d bosonic black hole is obtained by eq. (4.4) with $\kappa = \kappa_B - 2 = \frac{1}{4}$, $j = -\frac{1}{2} + i\frac{P}{2}$, and the on shell condition $m = \mp\bar{m} = i\frac{3P}{2}$. References [4, 12] do not have the κ-dependent phase factor in (4.4), since they analyze only the classical limit (equivalent to $\kappa \to \infty$ in (4.4)).

[25] Wave functions scattered behind the singularity of bosonic 2-d black holes were considered recently in [12]. In the charged cases it was found that such wave functions are partly transmitted through the singularity. However, in the uncharged case the wave functions are fully reflected from the singularity [4, 12]. On the other hand, in type 0B we find that, unlike the bosonic case and the NS-NS scalars in type 0, the R-R scalars are not fully reflected from the singularity. Instead, they are *fully transmitted* through the singularity.

scalar field is scattered from the horizon. In eq. (4.20), the dispersion relation is $E = -P$ ($E = -i(m+n-\bar{m}-\bar{n}) = -2i(m+1/2) = -ik$, $P = -2i(j+1/2) = ik$ in $\tilde{\mathbf{S}}^{-}(k)$; see appendix D). Hence, (4.20) is the inverse of the reflection coefficient; it satisfies:

$$|R_h(P)|^{-1} = \frac{\sinh(\pi P)}{\pi P} > 1\,, \tag{5.5}$$

(see $|(4.20)|$). The reflection coefficient itself is obtained for the correlator of scattering waves with $E = P$ (see appendix D). The other possibility (the analytic continuation of (4.19)) is $R_h(P) = 0$. The latter is supported by classical space-time considerations.[26]

6. A matrix model dual — a conjecture

In this section we speculate on an open/closed string duality between the type 0 $\mathrm{SL}(2)_{\frac{1}{2}}/\,\mathrm{U}(1)$ black hole, and the large N decoupled quantum matrix model on its (unstable) D-branes. Our conjecture follows a similar conjecture for the type 0 string theory on Super-Liouville $\times$ $\hat{c} = 1$ matter [8, 9]. Hence, we first review the main essence of the conjecture of [8, 9].

First, it is believed [22, 23, 24] that the bosonic closed string theory on Liouville $\times$ $c = 1$ matter is dual to the matrix quantum mechanics on the decoupled theory of its ZZ (1,1) branes [25]. The ZZ branes are D-branes localized in the strong coupling regime of the Liouville scalar ϕ (which has a linear dilaton). Their back reaction is expected to turn on the Liouville potential. The decoupled quantum mechanics in the large-N limit of such D-branes is conjectured [22, 23, 24] to be the "old" matrix model (for a review, see [26, 27]) of the 2-d bosonic string.

The ZZ branes are not stable. The open string tachyon potential has a local maximum. One side of this maximum is expected to have a local minimum while the other side is not bounded from below (see [28] and references therein). Hence, the matrix model dual is unstable non-perturbatively. This is reflected in the fact that the bosonic matrix eigenvalues act as free fermions in an inverted harmonic oscillator potential, which fill only one side of the potential up to a certain Fermi sea level. This potential is related to the open string tachyon potential, which turns in the double scaling limit (leading to the large-N matrix model) into the inverted harmonic oscillator potential.

A matrix model whose eigenvalues act as free fermions which fill the inverted harmonic oscillator potential symmetrically on both sides is conjectured [8, 9] to be the open string dual of the type 0B closed string theory on Super-Liouville $\times$ $\hat{c} = 1$ matter. Now, the inverted harmonic oscillator potential is associated with the open string tachyon potential of unstable D-branes in the two dimensional type 0 fermionic string. This potential is symmetric around its local maximum and has, in particular, local minima on both sides (see [28] and references therein). Hence, in the double scaling limit one obtains a stable vacuum where the eigenvalues fill the two sides of the potential to the same Fermi sea level.

[26] The R-R momentum modes should be described by a massless scalar which does not couple to the dilaton. Since the massless Laplace equation in two space-time dimensions is conformally invariant (upon taking into account the regularity conditions on the horizon), we see that the reflection coefficient should vanish.

A main ingredient in checking this duality is a simple relation between correlators in the 2-d type 0 theory and the 2-d bosonic string. An intuitive reason to this similarity is revealed by inspecting the Liouville potential and the on-shell vertex operators in both cases. In the 2-d bosonic string theory the Liouville potential is:

$$\mathcal{L}'_B = \mu_B e^{-\sqrt{2}\phi} = \mu_B e^{-\frac{Q_B}{2}\phi}\,, \qquad Q_B = 2\sqrt{2}\,, \tag{6.1}$$

where ϕ is the Liouville scalar field with a background charge Q_B. On-shell vertex operators are:

$$T_B^{\pm}(k) = e^{ikx} e^{(-\frac{Q_B}{2}\pm k)\phi}\,, \qquad Q_B = 2\sqrt{2}\,, \tag{6.2}$$

where x is the $c = 1$ scalar.

On the other hand, in the 2-d type 0 theory the Super-Liouville potential is:

$$\mathcal{L}' = \mu \int d^2\theta e^{-\Phi} = \mu \int d^2\theta e^{-\frac{Q}{2}\Phi}\,, \qquad Q = 2\,, \tag{6.3}$$

where Φ is a scalar superfield with a background charge Q, whose physical components are ϕ and ψ_ϕ. The on-shell vertex operators (in the NS sector) are:

$$T_{NS}^{\pm}(k) = e^{-\varphi} e^{ikx} e^{(-\frac{Q}{2}\pm k)\phi}\,, \qquad Q = 2\,. \tag{6.4}$$

The similarity between (6.1), (6.2) and (6.3), (6.4), respectively, leads [14] to correlators in the type 0 theory which are closely related to those in the bosonic theory: they are the same, up to leg factors and a rescaling $P \to \sqrt{2}P_B$ (the $\sqrt{2}$ originating from the ratio $Q_B/Q = \sqrt{2}$).

Another 2-d bosonic, closed string theory which has a matrix model dual is the bosonic string on sine-Liouville [29]. Sine-Liouville is a theory of a real scalar ϕ with a background charge Q_B, a compact scalar x on a circle with radius R, and a potential:[27]

$$\mathcal{L}'_{sl} = \lambda_{sl} e^{\frac{R-2}{\sqrt{2}}\phi} e^{i\frac{R}{\sqrt{2}}(x(z)-\bar{x}(\bar{z}))} + \text{c.c.} \tag{6.5}$$

As in Liouville $\times$ $c = 1$ matter, the Virasoro central charge of sine-Liouville is

$$c_{sl} = 2 + 3Q_B^2\,, \tag{6.6}$$

and hence, criticality of the bosonic string implies

$$c_{sl} = 26 \quad \Leftrightarrow \quad Q_B = 2\sqrt{2}\,. \tag{6.7}$$

In this case, $\mathcal{L}'_{sl}$ in (6.5) is marginal:

$$h(\mathcal{L}'_{sl}) = \frac{R^2}{4} - \frac{1}{2}\frac{R-2}{\sqrt{2}}\left(\frac{R-2}{\sqrt{2}} + 2\sqrt{2}\right) = 1\,. \tag{6.8}$$

[27]We choose $\alpha' = 1$, hence we normalize R as in [29], such that $(k, \bar{k}) = \frac{1}{\sqrt{2}}(\frac{p}{R} + wR, \frac{p}{R} - wR)$, though unlike [29] we work with canonically normalized, dimensionless scalars ϕ and x, as in (2.33), namely, vertex operators on the circle are $e^{i(kx+\bar{k}\bar{x})}$ with $h(e^{ikx}) = \frac{k^2}{2}$. The self-dual radius is $R/\sqrt{\alpha'} = 1$, though the $R \to \alpha'/R$ T-duality is broken in the presence of the winding mode condensate (the sine-Liouville potential).

The matrix model dual of this string theory is studied in [29]. Again, the matrix eigenvalues act as free fermions in an inverted harmonic oscillator potential, but this time the eigenvalues dynamics involves not only the scalar representation of $\mathrm{SU}(N)$, but also higher representations. In any case, as in the "old" matrix model, discussed above, the eigenvalues fill only one side of the potential.

We finally return to the fermionic string on $\mathrm{SL}(2)/\mathrm{U}(1)$. The SCFT on the $\mathrm{SL}(2)_\kappa/\mathrm{U}(1)$ cigar is T-dual to the $N=2$ Liouville theory [10, 19, 30, 31]. The $N=2$ Liouville theory has a scalar ϕ with a background charge Q, a compact scalar x:

$$x \simeq x + 2\pi\sqrt{\frac{2}{\alpha'}}r\,, \qquad r = \frac{\sqrt{2\alpha'}}{Q}\,, \tag{6.9}$$

and a superpotential:

$$\mathcal{L}'_{N=2} = \lambda \int d^2\theta e^{-\frac{1}{Q}(\Phi+i\tilde{X})} + c.c\,, \qquad \tilde{X} \equiv \bar{X}(\bar{z}) - X(z)\,. \tag{6.10}$$

Here Φ is the superfield with physical components ϕ and ψ_ϕ, and X is a scalar superfield with physical components x and ψ_x, where ϕ and x are given in (2.33), (2.34), and Q and κ are related as in (2.34). As before, in a critical fermionic string on $N=2$ Liouville:

$$c_{N=2} = 15 \quad \Leftrightarrow \quad Q = 2 \quad \left(\kappa = \frac{1}{2}\right). \tag{6.11}$$

In this case,

$$\mathcal{L}'_{N=2}(Q=2) = \lambda \int d^2\theta e^{-\frac{1}{2}(\Phi+i\tilde{X})} + \text{c.c.} = \lambda \int d^2\theta e^{-\frac{Q}{4}(\Phi+i\tilde{X})} + \text{c.c.}\,, \tag{6.12}$$

where the compactification radius of x is $r/\sqrt{\alpha'} = 1/\sqrt{2}$ (6.9) (in which case the $c=1$ CFT of x is equivalent to the theory of a free "Dirac fermion").

Following similar ideas to those in [8, 9], discussed above, we are led to conjecture that the matrix model dual of type 0B string theory on $N=2$ Liouville is the symmetric potential version of the KKK matrix model [29] at the self-dual radius $R/\sqrt{\alpha'} = 1$. Its non-perturbatively stable vacuum consists of matrix eigenvalues acting as free fermions, which fill both sides of the potential in a symmetric way. The reason that we have picked up the $R/\sqrt{\alpha'} = 1$ theory is because at this value of R the sine-Liouville potential (6.5) is:

$$\mathcal{L}'_{sl} = \lambda_{sl} e^{-\frac{\sqrt{2}}{2}(\phi+i\tilde{x})} + \text{c.c.} = \lambda_{sl} e^{-\frac{Q_B}{4}(\phi+i\tilde{x})} + \text{c.c.}\,, \qquad \tilde{x} \equiv \bar{x}(\bar{z}) - x(z)\,, \quad Q_B = 2\sqrt{2}\,. \tag{6.13}$$

Moreover, the asymptotic behavior of the on-shell vertex operators in the weak coupling regime of sine-Liouville and $N=2$ Liouville is the same as (6.2) and (6.4), respectively. Comparing (6.13) to (6.12) we see that the bosonic string theory on the $R/\sqrt{\alpha'} = 1$ sine-Liouville theory has similar correlators to those of the type 0B string on $N=2$ Liouville (up to leg factors and $P \to \sqrt{2}P_B$, originated from the ratio: $Q_B/Q = \sqrt{2}$).

Since the $N=2$ Liouville is T-dual to the SCFT on $\mathrm{SL}(2)/\mathrm{U}(1)$, we are finally led to our main conjecture: *the type 0A string theory on the* $\mathrm{SL}(2)_{\frac{1}{2}}/\mathrm{U}(1)$ *black hole is dual to the* $R/\sqrt{\alpha'}=1$ *KKK matrix model [29] with a symmetric potential and eigenvalues on both sides of its maximum.*

A few comments are in order:

(a) We further conjecture that this dual matrix model is the decoupled theory on $N\to\infty$ D-branes localized near the tip of the $\mathrm{SL}(2)/\mathrm{U}(1)$ cigar. These are the analogs of the ZZ branes discussed above [32, 33]. It is clear that they must exist for various reasons. One is that there are consistent configurations of D-branes stretched between NS5-branes (for a review, see [34]), and the near horizon limit of certain distributions of NS5-branes gives the $\mathrm{SL}(2)/\mathrm{U}(1)$ SCFT [10, 19]. A D-brane stretched between two such NS5-branes must turn into a D-brane localized near the tip of the cigar. Another reason is that localized D-branes in[28] $AdS_3\simeq \tilde{SL}(2)$ were found in [32, 33], following a similar route to ZZ [25]. By taking the "Fourier transform" (as in [10, 19]) of the closed string 1-p-f on the disc, computed in [32, 33], and inducing the result to the $\mathrm{SL}(2)/\mathrm{U}(1)$ quotient CFT, we find the analog of the ZZ brane construction in the cigar CFT.

(b) The sine-Liouville theory at the self-dual radius $R/\sqrt{\alpha'}=1$ is not the one dual to the $\mathrm{SL}(2)_{\frac{9}{4}}/\mathrm{U}(1)$ bosonic 2-d black hole. The latter is conjectured to be dual to sine-Liouville at $R/\sqrt{\alpha'}=3/2$ [29]. One can see that the matrix model dual to type 0 on the 2-d black hole cannot be the symmetric version of the KKK matrix model dual to the 2-d bosonic black hole by inspecting eq. (5.4): the on-shell condition in the bosonic case is $m=\pm\bar{m}=i\frac{3}{2}P$ while the on-shell condition in type 0 is $m=\pm\bar{m}=i\frac{P}{2}$ (the SL(2) levels are also different: $\kappa=1/2$ in type 0 versus $\kappa\equiv\kappa_B-2=1/4$ in the bosonic case). These lead to rather different correlators (4.4) in $\mathrm{SL}(2)/\mathrm{U}(1)$ (for instance, compare (5.4) to (4.12), (4.9) and (5.1)).

(c) The $R/\sqrt{\alpha'}=1$ point (the self-dual radius) in the investigation of [29] is rather special. The methods used in [29] break down precisely as $R/\sqrt{\alpha'}$ is decreased below 1. Hence, it would be interesting to reconsider the matrix model at this special point.

(d) Our conjecture should of course be supported by more flesh, similar to [9]. For instance, it would be nice to prove the analog of eq. (3.47) in [9] (the factorization of the correlators of certain linear combinations $T_{L,R}$ of our $\mathbf{T}$ and $\mathbf{S}$), to find the ground ring in the type 0 2-d black hole (or $N=2$ Liouville) and the λ-dependent analog of eq. (3.39) in [9], and to compute the torus partition function (though it would make sense to do it in the models considered in (e), which depend on more parameters).

[28] Actually, in H_3^+ — the euclidean continuation of SL(2).

(*e*) It would be interesting to study the 2-d type 0 string theory on $R_\phi \times S^1_x$ (2.33) with a more generic superpotential:

$$\int d^2\theta \left(\mu e^{-\Phi} + \lambda e^{\frac{R-2}{2}\Phi - i\frac{R}{2}\tilde{X}} + \text{c.c.}\right). \tag{6.14}$$

It is likely that it is dual to a symmetric version of the (μ, λ, R)-dependent family of KKK matrix models.[29]

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A. BRST invariance

In this appendix we will check the BRST invariance of the Ramond vertex operators in the $-1/2$ and $-3/2$ pictures. Let us begin with the asymptotic regime, where the $N = 2$ superconformal matter system is given by

$$\begin{aligned}
T_m &= -\frac{1}{2}\partial\phi\partial\phi - \frac{1}{2}\psi_\phi\partial\psi_\phi - \frac{Q}{2}\partial^2\phi - \frac{1}{2}\partial x\partial x - \frac{1}{2}\psi_x\partial\psi_x\,,\\
G^\pm &= \frac{i}{2}(\psi_\phi \pm i\psi_x)\partial(\phi \mp ix) + \frac{i}{2}Q\partial(\psi_\phi \pm i\psi_x)\,,\\
J^{\mathrm{U}(1)} &= i\psi_x\psi_\phi + iQ\partial x\,, \qquad Q = 2\,.
\end{aligned} \tag{A.1}$$

The BRST operator can be written as

$$\begin{aligned}
Q_{BRST} &= Q_0 + Q_1 + Q_2\,,\\
Q_0 &= \oint cT(\phi, x, \psi_\phi, \psi_x, \beta, \gamma) + bc\partial c\,,\\
Q_1 &= -\oint \gamma G_m = -\oint e^{\varphi}\eta G_m\,,\\
Q_2 &= -\frac{1}{4}\oint b\gamma^2 = -\frac{1}{4}\oint be^{2\varphi}\eta^2\,,
\end{aligned} \tag{A.2}$$

where the matter $N = 1$ supercurrent G_m is

$$G_m = G^+ + G^- = i\psi_\phi\partial\phi + i\psi_x\partial x + iQ\partial\psi_\phi\,, \tag{A.3}$$

and $T(\phi, x, \psi_\phi, \psi_x, \beta, \gamma)$ is the sum of T_m and the stress-energy tensor for the (β, γ) system of the superconformal ghosts. Note that we further bosonize the latter as $\beta = e^{-\varphi}\partial\xi, \gamma =$

[29] There is a difficulty here (which applies both to (6.14) and (6.5)), pointed out to us by N. Seiberg: it is likely that there are only 2 independent truly marginal deformations of the cigar CFT (or its T-dual Liouville). The value of the asymptotic radius R is correlated with λ (as long as $\lambda \neq 0$). Thus one should clarify what is the KKK [29] string theory at $R/\sqrt{\alpha'} = 1$.

$e^{\varphi}\eta$, as usual, with $\varphi(z)\varphi(w) \sim -\log(z-w)$ and the fermionic (ξ, η) system of conformal dimension $(0,1)$ satisfying $\xi\xi \sim \eta\eta \sim 0$, $\xi(z)\eta(w) \sim \frac{1}{z-w}$.

The primaries of the Ramond sector in the SCFT (A.1), which satisfy the mass shell condition $h = 5/8$ in the $-1/2$ and $-3/2$ pictures, are:

$$\mathcal{S}^{\epsilon_1,\epsilon_2}(k) = e^{i\frac{\epsilon_2}{2}H'} e^{ikx} e^{(-1+\epsilon_1 k)\phi}\,, \qquad \epsilon_{1,2} = \pm 1\,, \tag{A.4}$$

where H' is given by (2.37):

$$\partial H' = \psi_x \psi_\phi \quad \Longleftrightarrow \quad e^{\pm iH'} = \frac{1}{\sqrt{2}}(\psi_\phi \pm i\psi_x)\,. \tag{A.5}$$

Using (A.5) and $Q = 2$, we can express

$$G_m = \frac{i}{\sqrt{2}}(e^{iH'} + e^{-iH'})\partial\phi + \frac{1}{\sqrt{2}}(e^{iH'} - e^{-iH'})\partial x + i\sqrt{2}\partial(e^{iH'} + e^{-iH'})\,. \tag{A.6}$$

Now one obtains:

$$\begin{aligned} &G_m(z)\mathcal{S}^{\epsilon_1,\epsilon_2}(k)(w) \sim \\ &\quad -\frac{ik}{\sqrt{2}} e^{(-1+\epsilon_1 k)\phi} e^{ikx} \left[\frac{(\epsilon_1+1)}{(z-w)^{1-\frac{\epsilon_2}{2}}} e^{i(\frac{\epsilon_2}{2}+1)H'} + \frac{(\epsilon_1-1)}{(z-w)^{1+\frac{\epsilon_2}{2}}} e^{i(\frac{\epsilon_2}{2}-1)H'} \right] + \cdots\,, \end{aligned} \tag{A.7}$$

where the "$\cdots$" stand for other terms with no $(z-w)^{-\frac{3}{2}}$ factors. Hence,

$$G_m(z)\mathcal{S}^{\epsilon_1,\epsilon_2}(k)(w) \sim \frac{-ik}{\sqrt{2}} \frac{\delta_{\epsilon_1,-\epsilon_2}\mathcal{S}^{\epsilon_1,-\epsilon_2}(k)(w)}{(z-w)^{\frac{3}{2}}} + \frac{\mathcal{O}}{(z-w)^{\frac{1}{2}}} + \cdots\,. \tag{A.8}$$

We want to check whether Q_{BRST} commutes with

$$V_q^{\epsilon_1,\epsilon_2}(k) \equiv e^{q\varphi}\mathcal{S}^{\epsilon_1,\epsilon_2}(k)\,, \qquad \epsilon_{1,2} = \pm 1\,, \qquad q = -\frac{1}{2}, -\frac{3}{2}\,. \tag{A.9}$$

From the form of Q_1 in (A.2) and (A.9), (A.8), it follows that in the $-\frac{1}{2}$ picture we should impose:

$$-\frac{1}{2}\ \text{picture:} \qquad \epsilon_1 = \epsilon_2\,, \tag{A.10}$$

for $k \neq 0$. On the other hand, in the $-3/2$ picture all the four states (A.9) are BRST closed. However, the states in (A.9) with $q = -3/2$, $\epsilon_1 = \epsilon_2$ and $k \neq 0$ are BRST exact, since they are given by

$$V_{-3/2}^{\epsilon_1,\epsilon_1}(k)(z) = \frac{i}{\sqrt{2}k}\left[Q_{BRST}, e^{-\frac{5\varphi}{2}}\partial\xi\mathcal{S}^{\epsilon_1,-\epsilon_1}(k)(z)\right]\,. \tag{A.11}$$

Note that again the Q_0 and Q_2 terms do not contribute, and the action of Q_1 gives the necessary simple pole. As a final check of the BRST-exact character of $V_{-3/2}^{\epsilon_1,\epsilon_1}(k)$ and the non-triviality of $V_{-3/2}^{\epsilon_1,-\epsilon_1}(k)$, we can apply the picture changing operator to both pairs of states, obtaining:

$$\begin{aligned} V_{-3/2}^{\epsilon_1,-\epsilon_1}(k) &\Longrightarrow \left[Q_{BRST}, \xi V_{-3/2}^{\epsilon_1,-\epsilon_1}(k)\right] = -i\sqrt{2}kV_{-1/2}^{\epsilon_1,\epsilon_1}(k)\,, \\ V_{-3/2}^{\epsilon_1,\epsilon_1}(k) &\Longrightarrow \left[Q_{BRST}, \xi V_{-3/2}^{\epsilon_1,\epsilon_1}(k)\right] = 0\,. \end{aligned} \tag{A.12}$$

Thus we see that the picture-changing operator maps the BRST-exact states to zero, and connects the non-trivial states of the $-3/2$ and $-1/2$ pictures. To summarize, in the $-3/2$ picture physical operators satisfy

$$-\frac{3}{2}\ \text{picture:} \qquad \epsilon_1 = -\epsilon_2 \,, \tag{A.13}$$

modulo BRST-exact operators with $\epsilon_1 = \epsilon_2$. In the exact SL(2)/ U(1) SCFT, the $N = 2$ supercurrents and R-current of the superconformal algebra are given, for instance, by the Kazama-Suzuki construction [35]:[30]

$$\begin{aligned} G^{\pm} &= \frac{1}{\sqrt{\kappa}}\psi^{\pm} j^{\mp} \,, \\ J_R &= i\partial H + \frac{2}{\kappa} J^3 = i\sqrt{\frac{c}{3}}\partial X_R \,, \end{aligned} \tag{A.14}$$

where the j^a and J^a, $a = \pm, 3$, are the bosonic and total currents, respectively, of the parent SL(2) SCFT, and ψ^a are the free fermions of SL(2) (see (2.1), (2.2), (2.3), (2.5)). Thus, to check BRST invariance one should compute the OPE between

$$\mathcal{S}^{\epsilon_1,\epsilon_2}(k) = V^{\epsilon_2 \frac{1}{2}}_{-\frac{1}{2}+\epsilon_1\frac{k}{2},\frac{k}{2}-\epsilon_2\frac{1}{2}} \,, \qquad \epsilon_{1,2} = \pm \,, \tag{A.15}$$

and the $N = 1$ supercurrent

$$G_m = \frac{1}{\sqrt{\kappa}}\left[\psi^+ j^- + \psi^- j^+\right] . \tag{A.16}$$

The result is the same as before ($\epsilon_1 = \epsilon_2$ in the $-1/2$ picture). This can be seen in the following way. One knows what is the action of $j^{\pm}$ and $\psi^{\pm}$ on the SL(2) primaries Φ_{jm} and e^{iH} (see (2.6)), respectively. Then using (2.18), (2.21) and the fact that $j^{\pm} = \tilde{j}^{\pm} e^{\pm\sqrt{\frac{2}{k+2}}x_3}$ with $\tilde{j}^{\pm} x_3 \sim 0$ (x_3 is defined in (2.12)), one finds the OPE between the combinations $\psi^{\pm} j^{\mp}$ in G_m and V^n_{jm} (2.22). The result (A.10) then follows for the $e^{-\frac{\varphi}{2}} V^n_{jm}$ given in (A.15). In the $-3/2$ picture all the four operators $e^{-\frac{3\varphi}{2}} V^n_{jm}$ in (A.15) are BRST-invariant (though two independent linear combinations of them are exact).[31]

B. 2-P-F in the NS-NS sector

Consider first the case $k > 0$ ($j = -1/2 + k/2$) and $m = -\bar{m}$. Inserting the appropriate values of $j, m, \bar{m}$ and $\kappa = 1/2$ into eq. (4.4), and using eq. (4.2), we find that:[32]

$$k = p \in \mathbb{Z}_+ \,, \quad m = -\bar{m} = \frac{p}{2} :$$

$$\langle \mathbf{T}^{--}(-p)\mathbf{T}^{+-}(p)\rangle = \langle V_{-\frac{1}{2}+\frac{p}{2},-\frac{p}{2},\frac{p}{2}} V_{-\frac{1}{2}+\frac{p}{2},\frac{p}{2},-\frac{p}{2}}\rangle = \left(\frac{\Gamma(1-2p)\Gamma(-p)}{\Gamma(1+2p)\Gamma(p)}\right)\left(\frac{\Gamma(\frac{1}{2}+p)}{\Gamma(\frac{1}{2})}\right)^2 . \tag{B.1}$$

[30] The $N = 2$ U(1) R-current J_R was given in (2.25) (see (2.10), (2.9)).

[31] Those can be obtained by inspecting the asymptotic behavior mentioned in footnote 8 combined with (A.13).

[32] Although the unitarity bound (2.43) actually does not allow $j = \frac{p-1}{2}$, and even though for such values of j there are (double) poles in the euclidean 2-p-f, we write it formally as we are interested in its analytic continuation below.

After analytic continuation $k = p \to iP$, $P \in R$, to the $1+1$ lorentzian black hole, we have:

$$R_h(P) \equiv \langle \mathbf{T}^{--}(-iP)\mathbf{T}^{+-}(iP)\rangle = \left(\frac{\Gamma(1-2iP)\Gamma(-iP)}{\Gamma(1+2iP)\Gamma(iP)}\right)\left(\frac{\Gamma(\frac{1}{2}+iP)}{\Gamma(\frac{1}{2})}\right)^2 . \tag{B.2}$$

Next, for $k > 0$ and $m = \bar{m}$:

$$\begin{aligned} k = \frac{w}{2} \in \frac{\mathbb{Z}_+}{2}, \qquad m &= \bar{m} = \frac{w}{4} : \\ \left\langle \mathbf{T}^{-+}\left(-\frac{w}{2}\right)\mathbf{T}^{++}\left(\frac{w}{2}\right)\right\rangle &= \langle V_{-\frac{1}{2}+\frac{w}{4},-\frac{w}{4},-\frac{w}{4}} V_{-\frac{1}{2}+\frac{w}{4},\frac{w}{4},\frac{w}{4}}\rangle \\ &= \left(\frac{\Gamma(1-w)\Gamma(-\frac{w}{2})}{\Gamma(1+w)\Gamma(\frac{w}{2})}\right)\frac{\Gamma(\frac{1}{2}+\frac{w}{2})}{\Gamma(\frac{1}{2}-\frac{w}{2})} . \end{aligned} \tag{B.3}$$

After analytic continuation $k = \frac{w}{2} \to iP$, $P \in R$, to the lorentzian black hole, we have:

$$R_s(P) \equiv \langle \mathbf{T}^{-+}(-iP)\mathbf{T}^{++}(iP)\rangle = \left(\frac{\Gamma(1-2iP)\Gamma(-iP)}{\Gamma(1+2iP)\Gamma(iP)}\right)\frac{\Gamma(\frac{1}{2}+iP)}{\Gamma(\frac{1}{2}-iP)} . \tag{B.4}$$

We have thus obtained the 2-p-f of positive momentum modes and positive winding modes on the euclidean cigar, and their continuation to the $1+1$ lorentzian black hole. We now turn to negative momenta and windings and their analytic continuation.

For $k < 0$ ($j = -1/2 - k/2$) and $m = -\bar{m}$ one finds:

$$\begin{aligned} k = p \in \mathbb{Z}_-, \quad m &= -\bar{m} = \frac{p}{2} : \\ \langle \mathbf{T}^{+-}(-p)\mathbf{T}^{--}(p)\rangle &= \langle V_{-\frac{1}{2}-\frac{p}{2},-\frac{p}{2},\frac{p}{2}} V_{-\frac{1}{2}-\frac{p}{2},\frac{p}{2},-\frac{p}{2}}\rangle \\ &= \left(\frac{\Gamma(1+2p)\Gamma(p)}{\Gamma(1-2p)\Gamma(-p)}\right)\left(\frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}+p)}\right)^2 . \end{aligned} \tag{B.5}$$

Its analytic continuation to the lorentzian quotient gives:

$$\langle \mathbf{T}^{+-}(-iP)\mathbf{T}^{--}(iP)\rangle = R_h^{-1}(P) , \tag{B.6}$$

where $R_h(P)$ is given in (B.2).

Finally, for $k < 0$ and $m = \bar{m}$:

$$\begin{aligned} k = \frac{w}{2} \in \frac{\mathbb{Z}_-}{2}, \quad m &= \bar{m} = \frac{w}{4} : \\ \left\langle \mathbf{T}^{++}\left(-\frac{w}{2}\right)\mathbf{T}^{-+}\left(\frac{w}{2}\right)\right\rangle &= \langle V_{-\frac{1}{2}-\frac{w}{4},-\frac{w}{4},-\frac{w}{4}} V_{-\frac{1}{2}-\frac{w}{4},\frac{w}{4},\frac{w}{4}}\rangle \\ &= \left(\frac{\Gamma(1+w)\Gamma(\frac{w}{2})}{\Gamma(1-w)\Gamma(-\frac{w}{2})}\right)\frac{\Gamma(\frac{1}{2}-\frac{w}{2})}{\Gamma(\frac{1}{2}+\frac{w}{2})} , \end{aligned} \tag{B.7}$$

and its analytic continuation gives:

$$\langle \mathbf{T}^{++}(-iP)\mathbf{T}^{-+}(iP)\rangle = R_s(-P) = R_s^{-1}(P) , \tag{B.8}$$

where $R_s(P)$ is given in (B.4).

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C. 2-P-F in the R-R sector of type 0B

First of all, from eqs. (3.22) and (4.3) we see that a non-vanishing two point correlator in the R-R sector is between $\mathbf{S}_{-\frac{1}{2}}(k) \equiv \mathbf{S}(k)$ and (the "dagger" $\epsilon_2 \to -\epsilon_2$ of) its image in the $(-3/2, -3/2)$ picture (see appendix A):

$$\begin{aligned} \mathbf{S}^{\mp}_{-\frac{3}{2}}(k) &= e^{-\frac{3\varphi}{2}-\frac{3\bar{\varphi}}{2}} V^{\pm\frac{1}{2},\pm\frac{1}{2}}_{-\frac{1}{2}\mp\frac{k}{2},\frac{k}{2}\mp\frac{1}{2},\frac{k}{2}\mp\frac{1}{2}} \\ &= e^{-\frac{3\varphi}{2}-\frac{3\bar{\varphi}}{2}} V_{-\frac{1}{2}\mp\frac{k}{2},\frac{k}{2}\mp\frac{1}{2},\frac{k}{2}\mp\frac{1}{2}} e^{\frac{i}{\sqrt{5}}(2k\pm\frac{1}{2})(X_R(z)-\bar{X}_R(\bar{z}))} . \end{aligned} \tag{C.1}$$

Using the same methods as in the previous subsection, we find that for $k > 0$:

$$\begin{aligned} \langle \mathbf{S}^{-}_{-\frac{3}{2}}(-k)\mathbf{S}^{+}_{-\frac{1}{2}}(k)\rangle &= \langle V_{-\frac{1}{2}+\frac{k}{2},\frac{1}{2}-\frac{k}{2},\frac{1}{2}-\frac{k}{2}} V_{-\frac{1}{2}+\frac{k}{2},-\frac{1}{2}+\frac{k}{2},-\frac{1}{2}+\frac{k}{2}}\rangle \\ &= \frac{\Gamma(1-2k)\Gamma(-k)\Gamma(k)\Gamma(1)}{\Gamma(1+2k)\Gamma(k)\Gamma(1-k)\Gamma(0)} . \end{aligned} \tag{C.2}$$

For $k < 0$ we obtain:

$$\begin{aligned} \langle \mathbf{S}^{+}_{-\frac{3}{2}}(-k)\mathbf{S}^{-}_{-\frac{1}{2}}(k)\rangle &= \langle V_{-\frac{1}{2}-\frac{k}{2},-\frac{1}{2}-\frac{k}{2},-\frac{1}{2}-\frac{k}{2}} V_{-\frac{1}{2}-\frac{k}{2},\frac{1}{2}+\frac{k}{2},\frac{1}{2}+\frac{k}{2}}\rangle \\ &= \frac{\Gamma(1+2k)\Gamma(k)\Gamma(1)\Gamma(-k)}{\Gamma(1-2k)\Gamma(-k)\Gamma(0)\Gamma(1+k)} . \end{aligned} \tag{C.3}$$

Note that since here $m = \bar{m}$, namely, $k = w/2 \in \mathbb{Z}/2$, we actually obtain that (C.2) = (C.3) $= \frac{2(-)^w}{(|w|!)^2}$.[33] Due to the $\Gamma(0)$ in the denominator of eqs. (C.2), (C.3), it does not make sense to analytically continue to lorentzian space.[34]

Similarly, one finds that for $k > 0$:

$$\begin{aligned} \langle \mathbf{S}^{-}_{-\frac{1}{2}}(-k)\mathbf{S}^{+}_{-\frac{3}{2}}(k)\rangle &= \langle V_{-\frac{1}{2}+\frac{k}{2},-\frac{1}{2}-\frac{k}{2},-\frac{1}{2}-\frac{k}{2}} V_{-\frac{1}{2}+\frac{k}{2},\frac{1}{2}+\frac{k}{2},\frac{1}{2}+\frac{k}{2}}\rangle \\ &= \left(\frac{\Gamma(1+2k)\Gamma(k)\Gamma(1)\Gamma(-k)}{\Gamma(1-2k)\Gamma(-k)\Gamma(0)\Gamma(1+k)}\right)^{-1} , \end{aligned} \tag{C.4}$$

and for $k < 0$:

$$\begin{aligned} \langle \mathbf{S}^{+}_{-\frac{1}{2}}(-k)\mathbf{S}^{-}_{-\frac{3}{2}}(k)\rangle &= \langle V_{-\frac{1}{2}-\frac{k}{2},\frac{1}{2}-\frac{k}{2},\frac{1}{2}-\frac{k}{2}} V_{-\frac{1}{2}-\frac{k}{2},-\frac{1}{2}+\frac{k}{2},-\frac{1}{2}+\frac{k}{2}}\rangle \\ &= \left(\frac{\Gamma(1-2k)\Gamma(-k)\Gamma(k)\Gamma(1)}{\Gamma(1+2k)\Gamma(k)\Gamma(1-k)\Gamma(0)}\right)^{-1} . \end{aligned} \tag{C.5}$$

Note that (C.4) and (C.5) are the inverse of (C.3) and (C.2), respectively. The comments below eq. (C.3) thus apply here as well.

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[33] Use $\Gamma(x+1) = x\Gamma(x)$ and $\Gamma(-n+\epsilon) = \frac{(-)^n}{\epsilon n!} + O(1)$, $n = 0, 1, 2, \ldots$, to obtain this result.

[34] One obtains that in both cases the lorentzian 2-p-f for the continuous series is $R_s(P) = 0$ for all $P \in R$ (note that the analytic continuation $k \to iP$ is done before we write $\Gamma(k)$ in terms of $k!$, which is valid for $k \in \mathbb{Z}$ but not for $k \in iR$).

D. 2-P-F in the R-R sector of type 0A

We now have the two point correlator between $\tilde{\mathbf{S}}_{-\frac{1}{2}}(k) \equiv \tilde{\mathbf{S}}(k)$ (3.30) and $\tilde{\mathbf{S}}_{-\frac{3}{2}}(k)$. The latter is the same as $\tilde{\mathbf{S}}(k)$ except for the changes $e^{-\frac{\varphi}{2}-\frac{\bar{\varphi}}{2}} \to e^{-\frac{3\varphi}{2}-\frac{3\bar{\varphi}}{2}}$ (and $j \to -(j+1)$ if $\epsilon_2 = -\epsilon_1$; see appendix A).

We find that for $k > 0$:

$$\begin{aligned} \langle \tilde{\mathbf{S}}^{-}_{-\frac{3}{2}}(-k)\tilde{\mathbf{S}}^{+}_{-\frac{1}{2}}(k)\rangle &= \langle V_{-\frac{1}{2}+\frac{k}{2},\frac{1}{2}-\frac{k}{2},-\frac{1}{2}+\frac{k}{2}} V_{-\frac{1}{2}+\frac{k}{2},-\frac{1}{2}+\frac{k}{2},\frac{1}{2}-\frac{k}{2}}\rangle \\ &= \frac{\Gamma(1-2k)\Gamma(-k)}{\Gamma(1+2k)\Gamma(k)}\left(\frac{\Gamma(k)}{\Gamma(0)}\right)^2, \end{aligned} \tag{D.1}$$

and for $k < 0$ we obtain:

$$\begin{aligned} \langle \tilde{\mathbf{S}}^{+}_{-\frac{3}{2}}(-k)\tilde{\mathbf{S}}^{-}_{-\frac{1}{2}}(k)\rangle &= \langle V_{-\frac{1}{2}-\frac{k}{2},-\frac{1}{2}-\frac{k}{2},\frac{1}{2}+\frac{k}{2}} V_{-\frac{1}{2}-\frac{k}{2},\frac{1}{2}+\frac{k}{2},-\frac{1}{2}-\frac{k}{2}}\rangle \\ &= \frac{\Gamma(1+2k)\Gamma(k)}{\Gamma(1-2k)\Gamma(-k)}\left(\frac{\Gamma(1)}{\Gamma(1+k)}\right)^2. \end{aligned} \tag{D.2}$$

Note that since here $m = -\bar{m}$, namely, $k = p \in \mathbb{Z}$, we actually obtain that (D.1) = (D.2) $= \frac{2(-)^{p-1}}{(|2p|!)^2}$. The analytic continuation of (D.2) gives (for $P < 0$, since we choose the branch with $-i(j+1/2) > 0$):

$$(R_h(P))^{-1} = \frac{\Gamma(1+2iP)\Gamma(iP)}{\Gamma(1-2iP)\Gamma(-iP)}\left(\frac{\Gamma(1)}{\Gamma(1+iP)}\right)^2. \tag{D.3}$$

Similarly, one finds that for $k > 0$:

$$\begin{aligned} \langle \tilde{\mathbf{S}}^{-}_{-\frac{1}{2}}(-k)\tilde{\mathbf{S}}^{+}_{-\frac{3}{2}}(k)\rangle &= \langle V_{-\frac{1}{2}+\frac{k}{2},-\frac{1}{2}-\frac{k}{2},\frac{1}{2}+\frac{k}{2}} V_{-\frac{1}{2}+\frac{k}{2},\frac{1}{2}+\frac{k}{2},-\frac{1}{2}-\frac{k}{2}}\rangle \\ &= \left(\frac{\Gamma(1+2k)\Gamma(k)}{\Gamma(1-2k)\Gamma(-k)}\right)^{-1}\left(\frac{\Gamma(1)}{\Gamma(1+k)}\right)^{-2}, \end{aligned} \tag{D.4}$$

and for $k < 0$:

$$\begin{aligned} \langle \tilde{\mathbf{S}}^{+}_{-\frac{1}{2}}(-k)\tilde{\mathbf{S}}^{-}_{-\frac{3}{2}}(k)\rangle &= \langle V_{-\frac{1}{2}-\frac{k}{2},\frac{1}{2}-\frac{k}{2},-\frac{1}{2}+\frac{k}{2}} V_{-\frac{1}{2}-\frac{k}{2},-\frac{1}{2}+\frac{k}{2},\frac{1}{2}-\frac{k}{2}}\rangle \\ &= \left(\frac{\Gamma(1-2k)\Gamma(-k)}{\Gamma(1+2k)\Gamma(k)}\right)^{-1}\left(\frac{\Gamma(k)}{\Gamma(0)}\right)^{-2}. \end{aligned} \tag{D.5}$$

Note that (D.4) and (D.5) are the inverse of (D.2) and (D.1), respectively. In particular, the analytic continuation of (D.4) is $R_h(P)$ (for $P > 0$, since we choose the branch with $-i(j+1/2) > 0$), the inverse of (D.3).

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